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Radio wave absorption in a Chapman ionization layer produced in a non-isothermal atmosphere

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Abstract : Formulae for calculating non-deviative and total absorption arising in a Chapman ionization layer produced in a non-isothermal atmosphere with a constant scale height gradient for vertically incident radio waves reflecting from above the layer and from within the bottom-half of the layer have been derived. The quasi-longitudinal approximation of the Appleton-Hartree magneto-ionic theory is adopted in deriving the formulae. It is found that both the absorption and its solar zenith angle dependence are affected by the presence of a temperature gradient in the atmosphere. The formulae for non-deviative and total absorption reduce to the classical Appleton and Jaeger formulae for a Chapman ionization layer produced in an isothermal atmosphere.

Keywords : Ionosphere, radiowave absorption.

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1. Introduction

Appleton [1] showed that the non-deviative absorption (*i.e.*, absorption occurring in the region of the ionosphere characterized by a phase refractive index of magnitude close to unity) suffered by a vertically incident radio wave reflecting from above Chapman [2] ionization layer in an isothermal atmosphere with a single species neutral gas is given by

$$L = 2\int k\,dh = 4.133\,(\nu_0/c)\,H_0\left(\frac{f_0}{f \pm f_L}\right)^2 (\cos\chi)^{3/2} \quad (1)$$

for ordinary (+) and extra-ordinary (–) polarizations of the incident radio wave. In eq. (1)

- k = linear absorption coefficient,
- h = height,
- ν_0 = effective electron-neutral collision frequency at the height of unit optical depth for overhead Sun,
- χ = solar zenith angle,
- H_0 = scale height H at the height of unit optical depth for overhead Sun,
- f = sounding wave frequency $(\omega/2\pi)$,
- f_L = component of the electron gyro-frequency ω_B along the vertical $(\omega_L/2\pi)$,
- f_0 = electron plasma frequency in the Chapman layer at the height of unit optical depth for overhead Sun,
- c = speed of light in vacuum.

Subsequently, Jaeger [3] derived the formulae for total absorption suffered by a radio wave vertically incident in the Chapman layer produced in an isothermal and magnetic field-free atmosphere. For sounding waves of frequency f greater than the layer critical frequency f_c, he showed that the total absorption is given by

$$L = (\nu_0/c)\,H_0\,(f_0/f)^2\,\phi(f_c/f)\,(\cos\chi)^{3/2}, \quad (2)$$

where $\phi(f_c/f)$ is an integral function of the argument with an asymptotic value of 4.133 for f_c/f tending to zero *i.e.*, non-deviative absorption. For sounding waves of frequency f less than the layer critical frequency f_c, Jaeger [3] showed that the total absorption is given by

$$L=(\nu_0/c)H_0F_1(f/f_c)\cos\chi, \tag{3}$$

where the integral function $F_1(f/f_c)$ asymptotically approaches 4 for f/f_c tending to zero. Jaeger numerically evaluated the functions ϕ and F_1 for values of their respective arguments ranging from 0 to 0.98.

In reality, the terrestrial atmosphere is not isothermal. In the *D*-region, practically coincident with the mesosphere, the temperature decreases with altitude and in the *E*-region, it increases with altitude. In order to reconcile experimental determinations of absorption with those deduced from theory, it is necessary that the theory be modified by taking into consideration the temperature gradients in the atmosphere. The theory of non-deviative and total absorption arising in a Chapman ionization layer in a non-isothermal atmosphere with a constant scale height gradient is developed in the following sections.

2. Linear absorption coefficient

In the quasi-longitudinal approximation of the Appleton-Hartree magneto-ionic theory, the linear absorption coefficient k for a sounding wave of frequency f is given by

$$k=\frac{Ne^2\,\nu}{2m_e\,\epsilon_0\,c\mu\left\{\nu^2+(\omega\pm\omega_L)^2\right\}}, \tag{4}$$

where μ, the real part of the complex refractive index, in the same approximation, is given by

$$\mu=\left\{1-Ne^2/m_e\,\epsilon_0\,\omega^2(1\pm Y_L)\right\}^{1/2}, \quad (Y_L=f_L/f) \tag{5}$$

neglecting the collision term. The neutral gas species concentration n and the effective electron-neutral collision frequency ν in a non-isothermal atmosphere with a constant scale height gradient β are respectively given by

$$n=n_0\exp\left(-(1+\beta)z\right) \tag{6}$$

and

$$\nu=\nu_o\exp\left(-(1+\beta)z\right), \tag{7}$$

where z is the reduced height given by $\int_{h_0}^{h}\frac{dh}{H}$. The Chapman electron density altitude profile in such an atmosphere becomes

$$N=N_0\exp\tfrac{1}{2}\left\{1-(1+\beta)z-\mathrm{Ch}(\chi)\exp(-z)\right\}, \tag{8}$$

where the Chapman function $\mathrm{Ch}(\chi)$ reduces to $\sec(\chi)$ for values of $\chi<80°$. This equation reduces to eq. (328) of [4] when $\beta=0$ (for isothermal case) and $\mathrm{Ch}(\chi)=\sec(\chi)$. The Chapman layer ionization maximum N_m and critical sounding frequency f_c are derived from eq. (8) by partially differentiating it with respect to z and equating the derivative to zero. They are for $\chi<80°$,

$$N_m=N_0\left\{(1+\beta)^{(1+\beta)/2}\right\}\exp(-\beta/2)(\cos\chi)^{(1+\beta)/2} \tag{9}$$

and

$$f_c=f_0\left\{(1+\beta)^{(1+\beta)/4}\right\}\exp(-\beta/4)(\cos\chi)^{(1+\beta)/4}. \tag{10}$$

In an isothermal atmosphere, eqs. (9) and (10) reduce to the well-known equations

$$N_m=N_0\cos^{1/2}\chi \tag{11}$$

and

$$f_c=f_0\cos^{1/4}\chi. \tag{12}$$

Noting that $f_0=(e/2\pi)\left(N_0/m_e\,\epsilon_0\right)^{1/2}$ and using eqs. (8–10), eq. (5) reduces to

$$\mu=\left\{1-bt^{(1+\beta)/2}\exp(-t)\right\}^{1/2}, \tag{13}$$

where

$$t=\frac{1}{2}\sec\chi\exp(-z), \tag{14}$$

$$b=\left\{2/(1+\beta)\right\}^{(1+\beta)/2}\left(f_c/f\xi\right)^2\exp\left(\frac{1}{2}(1+\beta)\right) \tag{15}$$

and

$$\xi^2=(1\pm Y_L). \tag{16}$$

Using eqs. (8–10) and (13), eq. (4) for the linear absorption coefficient k becomes

$$k=\frac{2\left(\pi^2f_0^2/c\nu_0\right)\exp(1/2)(2\cos\chi)^{-(1+\beta)/2}\,t^{3(1+\beta)/2}\exp(-t)}{\left\{1-bt^{(1+\beta)/2}\exp(-t)^{1/2}\right\}\left\{t^{2(1+\beta)}+a^2\right\}} \tag{17}$$

wherein

$$a=\left\{(\omega\pm\omega_L)/\nu_0\,(2\cos\chi)^{1+\beta}\right\}. \tag{18}$$

The absorption suffered by the radio wave

$$L=2\int kdh=2\int Hkdz=2H_0\int k\exp(\beta z)dz$$
$$=-2^{(1-\beta)}H_0(\sec\chi)^{\beta}\int kt^{-(1+\beta)}dt \tag{19}$$

3. Absorption for sounding frequencies $f\xi > f_c$

For penetration of the Chapman layer by the sounding wave of frequency f such that $f\xi > f_c$, the limits of integration of (19) are $h \to 0$ to h_r (the height of reflection); $z \to -\infty$ to $+\infty$; $t \to \infty$ to 0. Using eq. (17) for k in eq. (19),

$$L = \frac{2^{3(1-\beta)/2}\pi^2 f_0^2 H_0}{c\nu_0(\cos\chi)^{(1+3\beta)/2}} \exp(1/2)$$

$$\int_0^\infty \frac{t^{(1+\beta)/2}\exp(-t)}{\left\{1 - bt^{(1+\beta)/2}\exp(-t)\right\}^{1/2}\left\{a^2 + t^{2(1+\beta)}\right\}} dt. \qquad (20)$$

If $f\xi >> f_c$ (*i.e.*, $f_c/f \to 0$), the absorption may be considered to be predominantly non-deviative ($\mu \sim 1$) so that eq. (20) reduces to

$$L = \frac{2^{3(1-\beta)/2}\pi^2 f_0^2 H_0}{c\nu_0} \exp(1/2)(\sec\chi)^{(1+\beta)/2}$$

$$\int_0^\infty \frac{t^{(1+\beta)/2}\exp(-t)}{\left(a^2 + t^{2(1+\beta)}\right)} dt. \qquad (21)$$

Since the height of unit optical depth for overhead Sun is greater than 100 km for most of the ionizing radiations of relevance in the ionosphere, $\omega \pm \omega_l >> \nu_0$ for sounding frequencies used in A1 absorption measurements. Hence, it is reasonable to assume that $a >> t^{(1+\beta)}$ so that eq. (20) reduces to

$$L = \frac{2^{(3+\beta)/2} f_0^2 H_0 \nu_0}{c(f \pm f_L)^2} \exp(1/2)(\cos\chi)^{(3+\beta)/2}$$

$$\int_0^\infty \frac{t^{(1+\beta)/2}\exp(-t)}{\left\{1 - bt^{(1+\beta)/2}\exp(-t)\right\}^{1/2}} dt$$

$$= (\nu_0 H_0/c)\left(\frac{f_0}{f \pm f_L}\right)^2 \phi(f_c/f\xi)(\cos\chi)^{(3+\beta)/2}, \qquad (22)$$

where $\phi(f_c/f\xi) = \left\{2^{(3+\beta)}\exp(1)\right\}^{1/2}$

$$\int_0^\infty \frac{t^{(1+\beta)/2}\exp(-t)}{\left\{1 - bt^{(1+\beta)/2}\exp(-t)\right\}^{1/2}} dt \qquad (23)$$

Table 1. $\phi(f_c/f\xi)$ as function of the scale height gradient β.

$f_c/f\xi/\beta$	−0.2	−0.15	−0.1	−0.05	0.0	0.05	0.1	0.15	0.2
0	3.871	3.933	3.999	4.068	4.141	4.218	4.298	4.383	4.472
0.05	3.874	3.936	4.002	4.071	4.144	4.221	4.302	4.387	4.475
0.10	3.884	3.946	4.012	4.081	4.155	4.232	4.313	4.398	4.487
0.15	3.900	3.962	4.029	4.098	4.172	4.249	4.331	4.416	4.506
0.20	3.922	3.986	4.052	4.123	4.197	4.275	4.356	4.442	4.533
0.25	3.952	4.016	4.084	4.155	4.229	4.308	4.390	4.477	4.568
0.30	3.990	4.055	4.123	4.195	4.270	4.350	4.433	4.521	4.613
0.35	4.037	4.102	4.171	4.244	4.321	4.401	4.486	4.575	4.668
0.40	4.093	4.160	4.230	4.304	4.382	4.464	4.550	4.640	4.735
0.45	4.161	4.229	4.300	4.376	4.455	4.539	4.626	4.718	4.815
0.50	4.241	4.311	4.384	4.461	4.543	4.628	4.718	4.812	4.910
0.55	4.337	4.409	4.484	4.564	4.647	4.735	4.827	4.923	5.025
0.60	4.453	4.527	4.605	4.687	4.773	4.863	4.958	5.058	5.162
0.65	4.593	4.670	4.751	4.836	4.926	5.019	5.118	5.221	5.329
0.70	4.765	4.846	4.931	5.02	5.114	5.212	5.314	5.422	5.535
0.75	4.983	5.068	5.158	5.252	5.351	5.454	5.562	5.675	5.794
0.80	5.266	5.358	5.454	5.555	5.661	5.771	5.886	6.007	6.132
0.85	5.657	5.759	5.865	5.976	6.091	6.210	6.334	6.464	6.598
0.90	6.254	6.375	6.498	6.624	6.753	6.885	7.020	7.160	7.305
0.92	6.607	6.741	6.875	7.011	7.147	7.285	7.425	7.569	7.716
0.94	7.089	7.244	7.397	7.547	7.693	7.837	7.979	8.122	8.268
0.95	7.411	7.584	7.751	7.912	8.064	8.210	8.351	8.489	8.629
0.96	7.823	8.024	8.214	8.389	8.549	8.695	8.829	8.958	9.084
0.98	9.236	9.587	9.898	10.152	10.337	10.457	10.523	10.556	10.576

Under the same approximation, eq. (21) reduces to

$$L = \left\{2^{(3+\beta)} \exp(1)\right\}^{1/2} (\nu_0 H_0/c) \left(\frac{f_0}{f \pm f_L}\right)^2$$

$$\Gamma((3+\beta)/2)(\cos\chi)^{(3+\beta)/2}. \quad (24)$$

If the Chapman layer is produced in an isothermal atmosphere, eq. (22) for total absorption reduces to

$$L = (\nu_0 H_0/c) \left(\frac{f_0}{f \pm f_L}\right)^2 \phi_0(f_c/f\xi)(\cos\chi)^{3/2}, \quad (25)$$

where

$$\phi_0 = 2(2\exp(1))^{1/2} \int_0 \frac{t^{1/2}\exp(-t)}{\left\{1 - b_0 t^{1/2}\exp(-t)\right\}^{1/2}} dt \quad (26)$$

and

$$b_0 = (2\exp(1))^{1/2} (f_c/f\xi)^2. \quad (27)$$

Eq. (24) for an isothermal atmosphere reduces to

$$L = 4.133\,(\nu_0 H_0/c) \left(\frac{f_0}{f \pm f_L}\right)^2 (\cos\chi)^{3/2}, \quad (28)$$

identical to eq. (1). Eq. (25) is identical to eq. (2) if it is remembered that in the Jaeger's treatment of the problem, the effect of the geo-magnetic field is ignored. The integral function $\phi(f_c/f\xi)$ of eq. (24) has been numerically evaluated for values of its argument 0 to 0.98 for varying values of β in the range –0.2 to 0.2 in steps of 0.05 by the 15 point Gauss-Laugerre quadrature method [5] and the results are presented in Table 1. In Table 2, we show the values of the ratio of non-deviative absorption for overhead Sun ($\chi = 0°$)

Table 2. Non-deviative absorption as a function of the scale height gradient β.

β	$2^{(3+\beta)/2}\exp(1/2)\Gamma((3+\beta)/2)$	$L_{0\beta}/L_{00}$
–0.20	3.808	0.921
–0.15	3.881	0.939
–0.10	3.955	0.957
–0.05	4.030	0.975
0	4.133	1
0.05	4.190	1.014
0.10	4.275	1.034
0.15	4.362	1.055
0.20	4.453	1.077

for $\beta \neq 0$ and $\beta = 0$ *i.e.*, $L_{0\beta}/L_{00}$ as a function of β as deduced from eq. (24).

4. Absorption for sounding frequences $f\xi < f_c$

When the product of the sounding frequency f and ξ, $f\xi$ is less than the critical frequency f_c, the wave propagates in the bottom-half of the Chapman layer from below ($z \to -\infty$) and undergoes reflection at $z = -z_1$ in the bottom-half of the layer. Or, if the wave is launched from above at $z = +\infty$, it propagates in the top-half of the layer undergoing reflection at $z = z_2$ above the layer peak. The expressions for t for waves propagating in the bottom and top halves of the Chapman layer at the points of reflection are given respectively by

$$t_1 = (1/2)\sec\chi\exp(z_1), \quad (29)$$

and

$$t_2 = (1/2)\sec\chi\exp(-z_2). \quad (30)$$

It is evident from these two equations that t_1 is greater than t_2. Since $\mu = 0$ at reflection, eq. (13) to (15) give

$$t^{(1+\beta)/2}\exp(-t) = 0.4289\,(f\xi/f_c)^2$$

$$\left\{(1+\beta)^{1+\beta}(2)^{-\beta}\exp(-\beta)\right\}^{1/2}. \quad (31)$$

For chosen values of $f\xi(< f_c)$ and β, it is seen that eq. (31) gives two roots of t; the greater of the two roots t_1 is relevant for propagation in the bottom-half of the Chapman layer. The total absorption of the wave is then given by eq. (20) with the lower limit of the integral changed from 0 to t_1 *i.e.*,

$$L = \frac{2^{3(1-\beta)/2}\pi^2 f_0^2 H_0}{c\nu_0(\cos(\chi))^{(1+3\beta)/2}} \exp(1/2)$$

$$\int^{\infty} \frac{t^{(1+\beta)/2}\exp(-t)}{\left\{1 - bt^{(1+\beta)/2}\exp(-t)\right\}^{1/2}\left\{a^2 + t^{2(1+\beta)}\right\}} dt. \quad (32)$$

The values of the lower limit of the integral t_1 for different values of $f\xi/f_c$ in the range 0 to 0.98 are determined for each value of β in the range –0.2 to 0.2 in steps of 0.05 and are shown in Table 3. Assuming once again that $\omega \pm \omega_L >> \nu_0$ (or $a >> t^{(1+\beta)}$), eq. (32) reduces to

$$L = (\nu_0 H_0/c)\left(\frac{f}{f \pm f_L}\right)\psi(f\xi/f_c)(\cos\chi), \quad (33)$$

where $$\psi(f\xi/f_c) = 2b\int_{t_1}^{\infty} \frac{t^{(1+\beta)/2}\exp(-t)}{\left\{1 - bt^{(1+\beta)/2}\exp(-t)\right\}^{1/2}} dt \quad (34)$$

$$= 4\int_0 d\mu + (1+\beta)b\int_{t_1}^{\infty} \frac{t^{-(1-\beta)/2}\exp(-t)}{\left\{1 - bt^{(1+\beta)/2}\exp(-t)\right\}^{1/2}}$$

$$= 4 + (1+\beta)b\int_{t_1}^{\infty} \frac{t^{-(1-\beta)/2}\exp(-t)}{\left\{1 - bt^{(1+\beta)/2}\exp(-t)\right\}^{1/2}} dt, \quad (35)$$

Table 3. Lower limit of t_1 of the integral function $(f\xi / f_c)$ as a function of the scale height gradient β.

$f_c / f / \beta$	−0.2	−0.15	−0.1	−0.05	0	0.05	0.1	0.15	0.2
0									
0.05	7.5675	7.6446	7.7205	7.7955	7.8695	7.9427	8.0150	8.0865	8.1573
0.10	6.0947	6.1670	6.2383	6.3086	6.3732	6.4468	6.5147	6.5819	6.6483
0.15	5.2219	5.2910	5.3590	5.4261	5.4925	5.5580	5.6228	5.6869	5.7503
0.20	4.5954	4.6618	4.7280	4.7917	4.8555	4.9185	4.9807	5.0424	5.1033
0.25	4.1039	4.1679	4.2310	4.2932	4.3548	4.4156	4.4756	4.5351	4.5939
0.30	3.6975	3.7594	3.8204	3.8806	3.9401	3.9989	4.0570	4.1145	4.1714
0.35	3.3497	3.4096	3.4686	3.5269	3.5845	3.6414	3.6977	3.7534	3.8085
0.40	3.0444	3.1024	3.1595	3.2160	3.2718	3.3269	3.3814	3.4354	3.4888
0.45	2.7712	2.8274	2.8827	2.9374	2.9915	3.0448	3.0977	3.1500	3.2017
0.50	2.5230	2.5773	2.6309	2.6838	2.7361	2.7878	2.8390	2.8896	2.9398
0.55	2.2944	2.3469	2.3987	2.4498	2.5005	2.5505	2.6000	2.6490	2.6976
0.60	2.0814	2.1321	2.1820	2.2315	2.2804	2.3287	2.3766	2.4240	2.4709
0.65	1.8807	1.9296	1.9777	2.0253	2.0725	2.1191	2.1653	2.2110	2.2563
0.70	1.6897	1.7366	1.7828	1.8286	1.8739	1.9187	1.9631	2.0072	2.0508
0.75	1.5055	1.5504	1.5946	1.6384	1.6819	1.7248	1.7674	1.8096	1.8514
0.80	1.3255	1.3682	1.4102	1.4520	1.4934	1.5343	1.5749	1.6152	1.6551
0.85	1.1461	1.1863	1.2260	1.2654	1.3045	1.3432	1.3816	1.4198	1.4576
0.90	0.9615	0.9989	1.0358	1.0725	1.1090	1.1451	1.1810	1.2167	1.2521
0.92	0.8839	0.9200	0.9556	0.9910	1.0263	1.0612	1.0956	1.1305	1.1648
0.94	0.8020	0.8365	0.8707	0.9047	0.9386	0.9722	1.0056	1.0388	1.0719
0.95	0.7585	0.7923	0.8256	0.8583	0.8920	0.9248	0.9574	0.9900	1.0223
0.96	0.7126	0.7454	0.7778	0.8102	0.8425	0.8745	0.9063	0.9381	0.9696
0.98	0.6076	0.6381	0.6683	0.6985	0.7287	0.7586	0.7884	0.8182	0.8479

Table 4. $\psi(f\xi / f_c)$ as a function of the scale height gradient β.

$f\xi / f_c / \beta$	−0.2	−0.15	−0.1	−0.05	0.0	0.05	0.1	0.15	0.2
0	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000
0.05	4.183	4.193	4.203	4.213	4.223	4.232	4.242	4.252	4.261
0.10	4.226	4.238	4.250	4.262	4.274	4.286	4.298	4.309	4.320
0.15	4.262	4.276	4.290	4.304	4.318	4.331	4.345	4.358	4.371
0.20	4.297	4.313	4.328	4.344	4.359	4.374	4.389	4.404	4.418
0.25	4.332	4.349	4.366	4.383	4.400	4.417	4.433	4.449	4.465
0.30	4.367	4.386	4.405	4.424	4.442	4.460	4.478	4.495	4.513
0.35	4.404	4.425	4.445	4.466	4.486	4.505	4.524	4.544	4.562
0.40	4.443	4.466	4.488	4.510	4.532	4.553	4.574	4.595	4.615
0.45	4.486	4.510	4.535	4.558	4.581	4.605	4.627	4.649	4.672
0.50	4.532	4.559	4.585	4.611	4.636	4.661	4.685	4.710	4.733

Table 4. *(Cont'd.).*

$f\xi/f_c/\beta$	−0.2	−0.15	−0.1	−0.05	0.0	0.05	0.1	0.15	0.2
0.55	4.584	4.613	4.641	4.669	4.696	4.723	4.750	4.776	4.802
0.60	4.642	4.673	4.705	4.735	4.764	4.794	4.822	4.851	4.879
0.65	4.709	4.743	4.777	4.810	4.842	4.874	4.905	4.936	4.967
0.70	4.788	4.826	4.863	4.899	4.934	4.969	5.003	5.037	5.070
0.75	4.884	4.925	4.966	5.006	5 045	5.084	5.121	5.158	5.195
0.80	5.005	5.051	5.097	5.141	5.184	5.227	5.269	5 310	5.351
0.85	5.165	5.218	5.270	5.321	5.370	5.419	5.466	5.513	5.559
0 90	5.401	5.463	5.525	5.584	5.641	5.699	5.754	5 809	5.863
0.92	5.535	5.602	5.668	5.733	5.795	5.857	5.918	5.976	6.034
0.94	5.711	5.785	5.858	5.929	5.997	6.065	6.131	6.197	6.260
0.95	5.824	5.902	5.980	6.055	6.126	6.198	6 269	6.337	6.405
0.96	5.964	6.047	6.131	6.210	6.287	6.363	6.439	6.511	6.584
0.98	6.402	6.502	6.602	6.698	6.790	6.882	6.972	7.059	7.144

where $d\mu = d\left[\left\{1 - bt^{(1+\beta)/2}\exp(-t)\right\}^{1/2}\right]$ from eq. (13). For a Chapman layer produced in an isothermal atmosphere, eq. (33) reduces to

$$L = (\nu_0 H_0/c)\left(\frac{f}{f \pm f_L}\right)\psi_0(f\xi/f_c)(\cos\chi), \qquad (36)$$

where

$$\psi_0(f\xi/f_c) = 4 + b_0\int_{t_1}^{\infty}\frac{t^{-1/2}\exp(-t)}{\left\{1 - b_0 t^{1/2}\exp(-t)\right\}^{1/2}}\,dt. \qquad (37)$$

In eqs. (36) and (37), the subscript 0 refers to $\beta = 0$. Eq. (36) is identical to eq. (3), wherein the effect of the geo-magnetic field has been ignored. The integral function $\psi(f\xi/f_c)$ of eq. (35) has been numerically evaluated for values of its argument 0 to 0.98 for each value of β in the range −0.2 to 0.2 in steps of 0.05 by the 15-point Gauss-Laugerre quadrature method and the results are presented in a tabular form in Table 4.

5. Discussion

Comparing eqs. (1 or 28) and (24) which give respectively the non-deviative absorption (*i.e.*, for $f\xi >> f_c$) occurring in a Chapman layer produced in isothermal and non-isothermal atmospheres, it is noted that both the absorption for overhead Sun ($\chi = 0°$) and solar zenith angle dependence are affected by the presence of a finite scale height gradient. Whereas the power exponent of cos χ in (1) is 3/2, it is $(3+\beta)/2$ in (24). It is thus possible to deduce the scale height gradient and thereby the temperature profile in the atmosphere using the plots of non-deviative absorption *versus* cos χ. In Table 2, we show the values of the ratio of non-deviative absorption for overhead Sun in non-isothermal and isothermal atmospheres $L_{0\beta}/L_{00}$ as a function of β. As β increases from −0.2 to 0.2, the ratio increases by about 16 percent.

A comparative sutdy of eqs. (22) and (25) which give the total absorption for sounding waves with $f\xi > f_c$ in non isothermal and isothermal atmospheres respectively shows that again the total absorption for overhead Sun and the solar zenith angle dependence are affected. At overhead Sun, the ratio of total absorption is ϕ/ϕ_0. As β varies from −0.2 to 0.2, ϕ increases by about 15 percent for any assumed value $f_c/f\xi$. From the tabulated data of Table 1, it is seen that $\phi/\phi_0 \gtrless 1$ for $\beta \gtrless 0$. The power exponents of cos χ are as in non-deviative absorption.

With regard to waves propagating in the bottom-half of the Chapman layer (*i.e.* $f\xi < f_c$), eqs. (33) and (36) show that the solar zenith angle dependence is independent of the scale height gradient. However, ψ is dependent on the scale height gradient as may be noted from the data shown in Table 4. At $f\xi/f_c = 0$, the total absorption is unaffected by the scale height gradient. But, as $f\xi/f_c$ increases, the effect of the scale height gradient becomes more and more pronounced

For $f\xi/f_c = 0.98$, the value of ψ increases by about 12 percent for an increase in β from –0.2 to 0.2.

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